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Scattering of surface waves by a conical island

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A model equation is derived which approximately describes the propagation of periodic surface waves in water of slowly varying depth. Numerical solutions to the model equation are obtained for the scattering of an incident plane wave by a conical island.

1. Introduction

The maximum run-ups of tsunamis (or seismic sea waves) on islands are often much greater than those on straight coastlines. Thus the modification of waves due to the seabed topography is an important aspect of the tsunamic phenomenon. Although tsunamis are dramatically nonlinear in their final run-up with amplitudes as high as 10 m, out at sea their amplitudes are considerably smaller and it is reasonable to assume that the topographic modification prior to run-up can be accurately described by linearized equations. Regrettably, the classical linear theory of water waves is not well suited to numerical calculations and there are only a few depth topographies that are amenable to analytic solution (Ursell 1952). Thus progress in the understanding of topographic modification has depended upon the use and invention of further simplifying approximations.

The most widely used approximation is that of linear shallow-water theory, in which the vertical structure of the waves is ignored. For this theory to be applicable to an oceanic situation it suffices that the period of the waves exceeds 5 min (Summerfield 1972). Another approximation is based upon the fact that the slope of the seabed is extremely small, rarely exceeding 0·01. According to the linear shallow-water theory and to the mild-slope theory, there are two mechanisms that contribute to the relatively large run-ups that occur at islands: refractive focusing (Eckart 1950; Keller 1958), and resonance of virtual trapped modes (Homma 1950; Shen, Meyer & Keller 1968). A major difference between the two theories, as presented in the literature, is that the linear shallow-water theory can yield detailed quantitative results concerning wave amplitudes (see Longuet-Higgins 1967, figures 9 and 10), while the mild-slope theory predicts only approximate resonance frequencies (Shen *et al.* 1968; Smith 1974).

The apparent shortcomings of the mild-slope theory can be attributed to the fact that it has always been used in conjunction with the methods of short-wave asymptotics. In §2 a model equation is proposed for which the geometrical-optics solution agrees to first order in the seabed slope with the solution for water waves derived by Keller (1958). Two tests of this model equation are presented in §3. Then in §§4 and 5 numerical solutions to the model equation are obtained for the scattering of a plane wave at a conical island. For shallow water the mild-slope equation reduces to the linear shallow-water wave equation. Thus we can check that the methods used in this paper yield results consistent with those of Lautenbacher (1970). Indeed, for irregularly shaped islands or sills the numerical methods for solving the linear shallow-water wave equation proposed by Vastano & Reid (1967) and by Lautenbacher (1970) would appear to be the best methods available for solving the mild-slope equation.

2. Model equations

The description of surface waves propagating over a seabed of mild slope that is presented by Keller (1958) and by Shen *et al.* (1968) is the first approximation in an asymptotic expansion for monochromatic waves. If first-order results suffice, then the asymptotic method of Keller & Rubinow (1960) permits the same description to be recovered from any model equation for which the local group and phase velocities at the frequency of interest are in agreement with the classical linear theory of water waves. One such model equation is

$$\nabla\cdot(c_p c_g \nabla\zeta) - \omega^2\left(\frac{c_p - c_g}{c_p}\right)\zeta - \frac{\partial^2\zeta}{\partial t^2} = 0. \tag{1}$$

Here ζ is the wave height, ∇ the horizontal gradient operator, ω the angular frequency of interest and c_p and c_g respectively are the local phase and group velocities of waves with frequency ω. It may readily be verified that the local dispersion relation for the model equation (1) has the required two-point contact at a frequency ω with the exact local dispersion relation for water waves.

For monochromatic waves of frequency ω equation (1) can be simplified to the mild-slope equation:

$$\nabla\cdot(p\nabla\zeta) + \omega^2 q\zeta = 0, \tag{2}$$

where p and q are the product and quotient respectively of the local group and phase velocities. If h is the local water depth and g is the gravitational acceleration, then

$$p = gh\frac{\tanh kh}{kh}\frac{1}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right), \tag{3a}$$

$$q = \tfrac{1}{2}(1 + 2kh/\sinh 2kh), \tag{3b}$$

where k is the positive real root of the transcendental equation

$$\omega^2 = gk\tanh kh. \tag{3c}$$

We note that (2) gives an exact description of the propagation of water waves in water of constant depth. A formal derivation of (2) is given in appendix A.

Eckart (1951) proposed a model equation of the same form as (2) but with

$$p_E = \frac{g^2}{\omega^2}\frac{1}{2}\left[1 - \exp\left(-\frac{2\omega^2 h}{g}\right)\right], \quad q_E = \frac{1}{2}\left[1 + \exp\left(-\frac{2\omega^2 h}{g}\right)\right]. \tag{4}$$

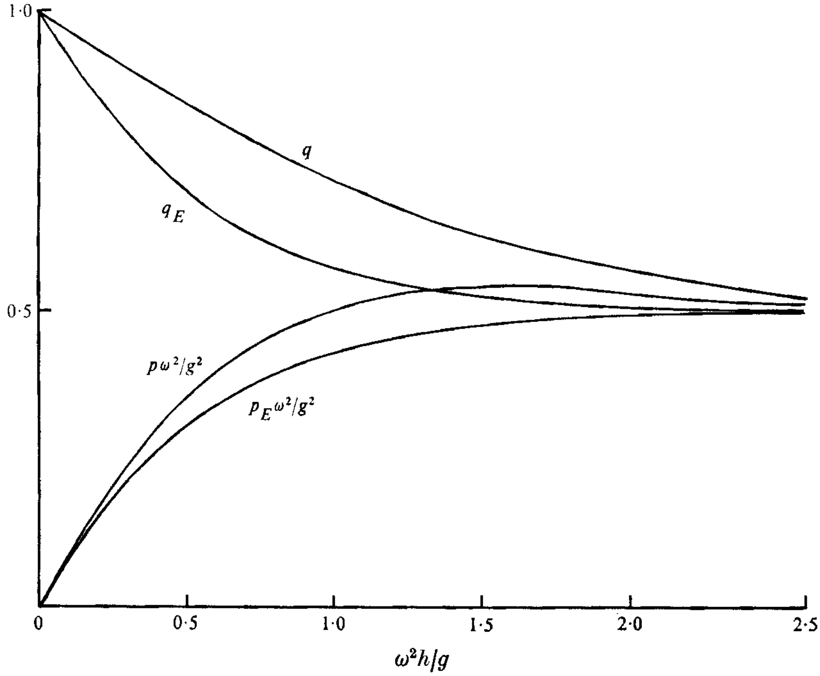

FIGURE 1. Comparison of the alternative p's and q's.

A comparison of the alternative equations (3) and (4) is presented in figure 1. Eckart (1951) gave several comparisons between the solutions to his model equation and the exact solutions for water of constant depth, and figure 1 corresponds to the test that caused him to criticize the accuracy of his model. If the waves are of high frequency or if the water is deep (i.e. $\omega^2h/g \gg 1$) then from both (3) and (4) we find that

$$\omega^2p/g^2 \sim \tfrac{1}{2}, \quad q \sim \tfrac{1}{2}.$$

These deep-water approximations are accurate to within 10 % in q and 5 % in p if ω^2h/g exceeds 2·2. Likewise, if the waves are of low frequency or if the water is shallow (i.e. $\omega^2h/g \ll 1$) then (3) and (4) are again in agreement:

$$p \sim gh, \quad q \sim 1,$$

and we recover the linear shallow-water equations for waves of frequency ω. Furthermore, it follows that, in this limit, (2) remains valid even if there are abrupt changes in depth. The shallow-water approximations are accurate to within 10 % in p and 5 % in q if ω^2h/g is less than 0·15.

3. Tests of the model equation

In practice the criterion by which a model equation is judged is not its asymptotic correctness, but its numerical accuracy for moderate slopes. An indication of the range of slopes for which the mild-slope equation is numerically accurate

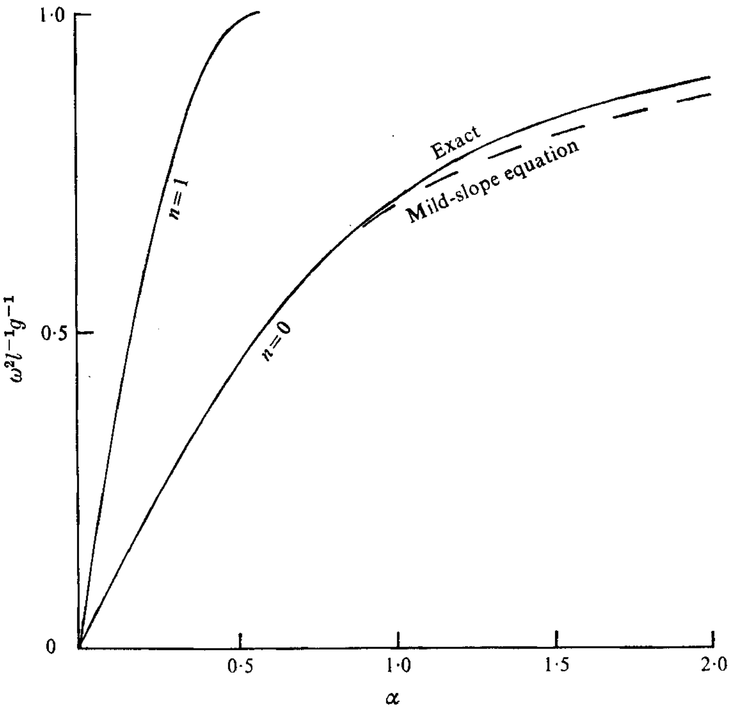

FIGURE 2. Dispersion relation for edge waves.

can be obtained by comparing the exact (Ursell 1952) and approximate dispersion relations for edge waves. For waves of longshore wavenumber l on a straight beach of depth $h = \alpha x$ equation (2) reduces to

$$\frac{d}{dx}\left(p\frac{d\zeta}{dx}\right) + (\omega^2 q - l^2 p)\,\zeta = 0.$$

The conditions for ζ to be an edge wave are that it is finite at the shoreline and decays exponentially far from the shore. It is straightforward to solve numerically the resulting Sturm–Liouville eigenvalue problem for l. Figure 2 compares these numerical solutions with the exact dispersion relation

$$\omega^2 l^{-1} g^{-1} = \sin\left[(2n+1)\tan^{-1}\alpha\right],$$

where n is the mode number. It is only for the lowest mode that the exact and approximate results are graphically distinguishable.

A more extreme test of the mild-slope equation is to apply it to a depth discontinuity. From (2) we can ascertain that the natural jump conditions are

$$\zeta_1 = \zeta_2, \quad p_1 \partial\zeta_1/\partial n = p_2 \partial\zeta_2/\partial n,$$

where the subscripts refer to the two sides of the depth discontinuity and $\partial/\partial n$ denotes the normal derivative. The reflexion and transmission of a wave propagating normal to a step can be represented by

$$\zeta = \begin{cases} e^{-ik_1 x} + Re^{ik_1 x} & \text{for } x < 0, \\ Te^{-ik_2 x} & \text{for } x > 0, \end{cases}$$

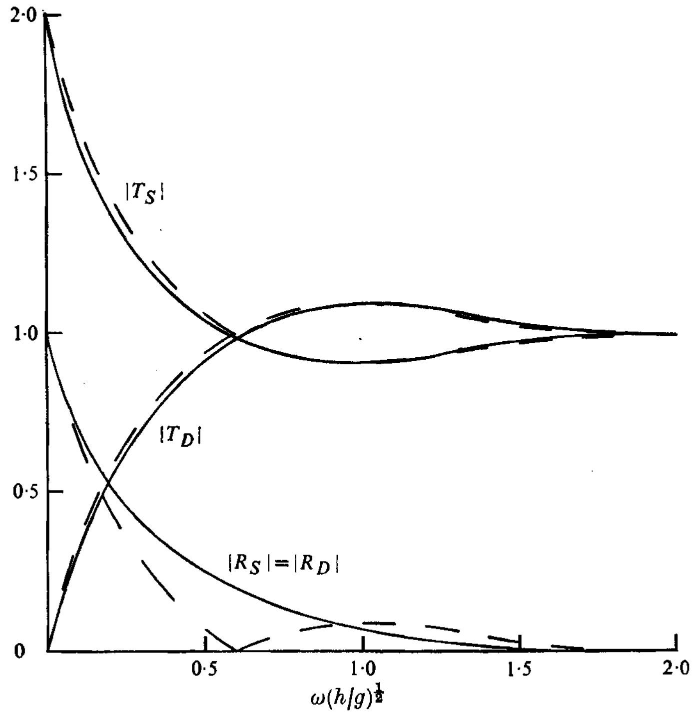

FIGURE 3. Exact (continuous curves) and approximate (dashed curves) reflexion and transmission coefficients. The subscripts D and S respectively refer to transmission into deep and shallow water.

where the reflexion and transmission coefficients R and T are given by

$$R = \frac{p_1k_1 - p_2k_2}{p_1k_1 + p_2k_2}, \quad T = \frac{2p_1k_1}{p_1k_1 + p_2k_2}.$$

For the special case in which one of h_1 and h_2 is infinite Newman (1965) has obtained numerical solutions of the exact linear equations for water waves. Figure 3 compares the exact and approximate values for R and T. Fortuitously, the model equation is most accurate precisely when the wave amplitudes are largest.

4. Scattering theory for circularly symmetric islands

Circular islands are important in that the compromise between mathematical simplicity and realistic geometry permits meaningful comparisons between different methods of studying the topographic modification of water waves (these methods include laboratory experiments, analog electrical circuits, numerical modelling and exact and approximate analytic calculations). Thus it is desirable that we should have an efficient means of solving the mild-slope equation (2) for the scattering of an incident plane wave by a circularly symmetric island. The change of dependent variable $\psi = p^{-\frac{1}{2}}\zeta$ turns (2) into a

Schrödinger equation. Hence the scattering theory for (2) can be regarded as being known (Morse & Feshbach 1953, chap. 12. However, for completeness we give a direct derivation of the results that are used.

The total wave height ζ comprises the incident wave $e^{i\kappa x}$ and a scattered wave ζ_s. If the time dependence is $e^{i\omega t}$ with ω positive, then the scattered wave is outgoing at large distances only if

$$\zeta_s \equiv \zeta - e^{i\kappa x} \sim f(\theta)\, e^{-i\kappa r} r^{-\frac{1}{2}} \quad \text{as} \quad r \to \infty,$$

where (r, θ) are the usual cylindrical polar co-ordinates.

The total wave will be represented as the linear combination

$$\zeta = A_0 e^{i\chi_0} \zeta_0(r) + 2 \sum_{m=1}^{\infty} A_m e^{i\chi_m} \zeta_m(r) \cos m\theta \tag{5}$$

of real solutions $\zeta_m(r) \cos m\theta$ of our model equation (2). The functions ζ_m satisfy the initial-value problem

$$\left.\begin{aligned} \frac{d}{dr}\left(rp\frac{d\zeta_m}{dr}\right) + \left(\omega^2 qr - \frac{m^2 p}{r}\right)\zeta_m = 0, \\ \zeta_m = 1 \ \text{ at the shoreline } r = a, \quad \lim_{r\to a} rp\, d\zeta_m/dr = 0. \end{aligned}\right\} \tag{6}$$

Now the incident wave has an angular representation in terms of Bessel functions:

$$\exp(i\kappa x) = J_0(\kappa r) + 2 \sum_{m=1}^{\infty} i^m J_m(\kappa r) \cos m\theta$$

(Morse & Feshbach 1953, p. 620), whose general term has the asymptotic form

$$i^m J_m(\kappa r) \sim (2/\pi\kappa r)^{\frac{1}{2}} \{\tfrac{1}{2}\exp[i(\kappa r - \tfrac{1}{4}\pi)] + (-1)^m \tfrac{1}{2} \exp[-i(\kappa r - \tfrac{1}{4}\pi)]\}.$$

In order to eliminate the radially incoming wave factor $\exp[i(\kappa r - \frac{1}{4}\pi)]$ in the asymptotic expansion for the scattered wave ζ_s it is necessary that as $r \to \infty$ the function $\zeta_m(r)$ has the asymptotic form

$$\zeta_m(r) \sim A_m^{-1} (2/\pi\kappa r)^{\frac{1}{2}} \cos(\kappa r - \tfrac{1}{4}\pi - \chi_m), \tag{7}$$

where A_m and χ_m are the same as in expression (5).

If the initial-value problem (6) is solved numerically then the asymptotic behaviour of ζ_m for large values of r is the least reliable feature of the solution. Thus it is desirable to replace (7) by a more stable means of evaluating A_m and χ_m. To do this, we employ the approximate (far field) Green's function

$$G_m(r, \bar{r}) = \tfrac{1}{2}\pi[J_m(\kappa\bar{r})\, Y_m'(\kappa r) - J_m(\kappa r)\, Y_m(\kappa\bar{r})] \quad (r < \bar{r}),$$

to derive an integral-equation version of the initial-value problem (6):

$$\begin{aligned} \frac{p(\bar{r})}{p_\infty}\zeta_m(\bar{r}) = J_m(\kappa\bar{r}) \bigg[\frac{\pi}{2}\frac{p(a)}{p_\infty}\kappa a\, Y_m(\kappa a) + \frac{\pi}{2}\int_a^{\bar{r}} \zeta_m(r) \bigg\{ \frac{\kappa}{p_\infty}\frac{dp}{dr} Y_m' \\ + \frac{\omega^2(qp_\infty - pq_\infty)}{p_\infty^2} Y_m \bigg\} r\, dr \bigg] - Y_m(\kappa\bar{r}) \bigg[\frac{\pi}{2}\frac{p(a)}{p_\infty}\kappa a J_m'(\kappa a) \\ + \frac{\pi}{2}\int_a^{\bar{r}} \zeta_m(r) \bigg\{ \frac{\kappa}{p_\infty}\frac{dp}{dr} J_m' + \frac{\omega^2(qp_\infty - pq_\infty)}{p_\infty^2} J_m \bigg\} r\, dr \bigg]. \end{aligned}$$

Hence, once we have an approximation to ζ_m over the island shelf we have an accurate means of determining the far field:

$$\zeta_m(\bar{r}) \sim J_m(\kappa\bar{r})\, L_1[\zeta_m] + Y_m(\kappa\bar{r})\, L_2[\zeta_m] \quad \text{as} \quad \bar{r} \to \infty,$$

where the linear functionals L_1 and L_2 are the limiting values of the corresponding multiplying factors in the integral equation. The required stable equations for A_m and χ_m are

$$A_m = (L_1^2 + L_2^2)^{-\frac{1}{2}}, \quad \chi_m = \tfrac{1}{2}m\pi + \arctan(L_2/L_1) + \tfrac{1}{2}(1 - \operatorname{sgn} L_1)\,\pi, \tag{8}$$

where $\arctan(L_2/L_1)$ is restricted to $(-\frac{1}{2}\pi, \frac{1}{2}\pi)$.

We note that the results (8) are strongly dependent upon the far-field wavenumber κ, through its occurrence in the argument of the Bessel functions. This can be expected to be a major source of error if an approximate dispersion relation, such as the shallow-water approximation, is used instead of the exact dispersion relation (3*c*).

5. Conical islands

In order to exemplify the use of (5), (6) and (8) we shall obtain numerical solutions for the scattering of plane periodic waves by conical islands.† There are two reasons for choosing this class of problems. First, the shallow-water calculations of Lautenbacher (1970) provide an independent check on the accuracy of the numerical methods used in the present paper. Second, laboratory experiments involving a conical island are currently being conducted by Barnard, Pritchard and Provis at the University of Essex, and it is hoped that the numerical results may assist the interpretation of the experimental results.

At a beach $p = 0$, and consequently the differential equation (6) is singular at the shoreline. Direct finite-difference calculations for the regular solution are ill conditioned near singular points. Indeed, Cohen & Jones (1974) have suggested that for such differential equations the solutions should be obtained by economized power-series expansions. Here we make the compromise of deriving a four-term Taylor series for ζ_m. This is used to start the solution (see appendix B), and away from the beach a standard finite-difference approximation is used (Gill 1951). The calculations were tested for self-consistency by changing the step lengths by factors of two.

Lautenbacher (1970) studies three conical islands, which he designates as Hawaii, Oahu and Small. For each island, graphs are presented of the maximum wave amplitude at the coast for incident waves of several different wavelengths. To reproduce these results by the methods of §4 we put $p = gh$ and $q = 1$ and evaluate the amplitudes A_m and phases χ_m for the first six modes. The maximum wave amplification at the coast is given by the equation

$$|\zeta| \doteqdot |e^{i\chi_0}A_0 + 2\sum_{m=1}^{6} e^{i\chi_m}A_m \cos m\theta|.$$

† Solutions for a parabolic island have been obtained by Jonsonn, Skovgaard and Brink-Kjaer (private communication).

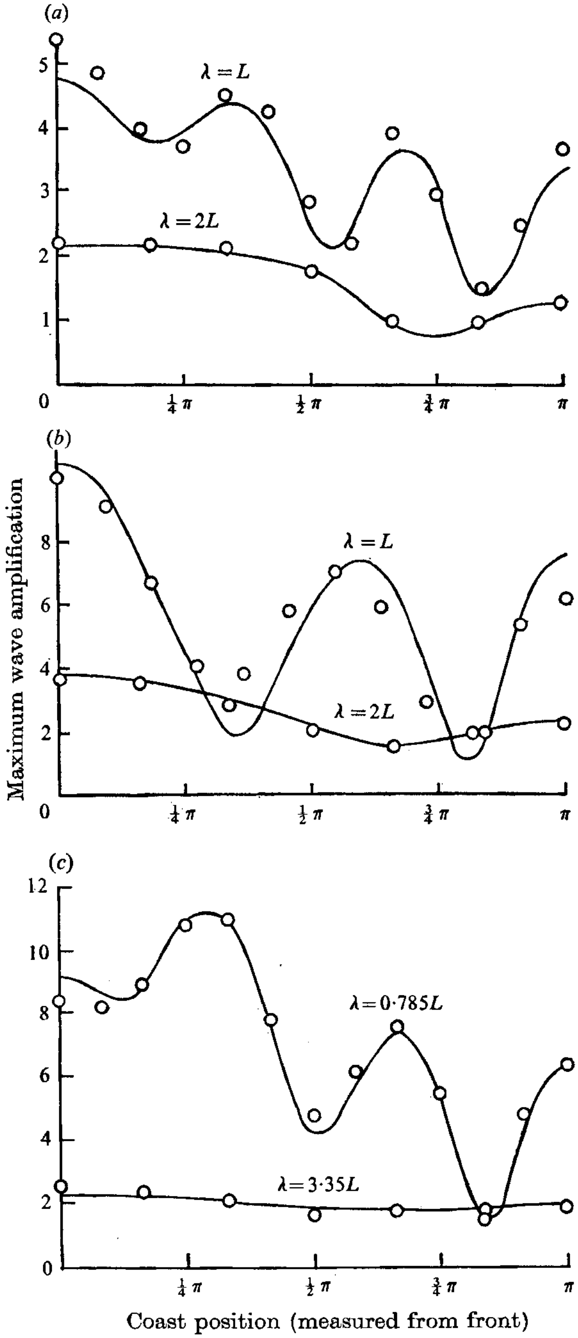

FIGURE 4. Comparison between finite-difference results (circles) and truncated modal expansions (continuous curves) for the maximum wave amplification at the coast. (*a*) Hawaii. (*b*) Oahu. (*c*) Small.

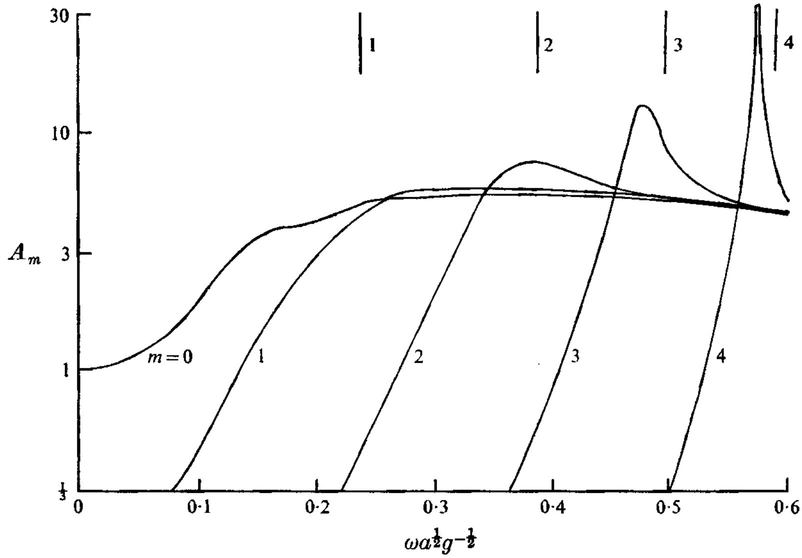

FIGURE 5. Frequency dependence of the amplitude factors A_m.

Figures 4(*a*)–(*c*) demonstrate the agreement between the truncated modal expansions of this paper and Lautenbacher's finite-difference calculations for the three islands. The parameter λ is the wavelength of the incident waves relative to the diameter of the island shelf.

The experiments being conducted by Barnard, Pritchard and Provis involve a cone of slope $\alpha = 0{\cdot}1$ with a shelf of radius 20 times that of the island. Figure 5 shows the computed frequency variation of the amplitude factors A_m for this particular cone geometry. The vertical lines are the resonance frequencies predicted for the edge wave with zero seaward mode number according to the short-wave asymptotic solution of Smith (1974):

$$\omega \sim (gm\alpha/a)^{\frac{1}{2}}\,[1 - \tfrac{1}{4}m^{-1} + O(m^{-2})].$$

There are two noteworthy features in figure 5. First, for each mode the amplitudes are well in excess of one once the frequency is higher than the first resonance frequency of the mode. These uniformly large amplitudes can be attributed to the mechanism of refractive focusing (Eckart 1950; Keller 1958). Second, for the first significant edge-wave resonance (in mode 3) the value of $\omega^2 h/g$ at the edge of the island cone is 0·43, which is well outside the range in which the shallow-water theory can be regarded as being an adequate approximation to the mild-slope theory.

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Appendix A. Formal derivation of the mild-slope equation

In the classical linear theory of water waves the velocity potential ϕ for waves with angular frequency ω satisfies the equations

$$\nabla^2\phi+\partial^2\phi/\partial z^2 = 0, \quad \partial\phi/\partial z = \omega^2\phi/g \quad \text{at} \quad z=0,$$
$$\partial\phi/\partial z+\nabla h.\nabla\phi = 0 \quad \text{at} \quad z=-h,$$

where z measures the height above the mean free surface and ∇ denotes the horizontal gradient operator. The method of separation of variables leads to consideration of the Sturm–Liouville problem

$$d^2w/dz^2+\lambda(h)\,w=0, \quad dw/dz=\omega^2 w/g \quad \text{at} \quad z=0, \quad dw/dz=0 \quad \text{at} \quad z=-h,$$

and the propagation of water waves is associated with the only eigenmode for which the eigenvalue $\lambda(h)$ is negative, i.e.

$$w_0(z;h)=\cosh\,[k(z+h)]/\cosh kh, \quad \lambda_0=-k^2,$$

where k satisfies the local dispersion relation (3c). Accordingly we represent ϕ by

$$\phi=a(\mathbf{x})\,w_0(z;h(\mathbf{x}))+\psi(\mathbf{x},z), \quad \text{where} \quad \int_{-h}^{0} w_0\psi\,dz=0.$$

Taking the w_0 component of the equations of motion (i.e. applying Green's identity to w_0 and ϕ) we obtain

$$\begin{aligned}\nabla.\left(\int_{-h}^{0} w_0^2 dz\nabla a\right)-\lambda_0\int_{-h}^{0} w_0^2 dz a &= -a\nabla^2 h\int_{-h}^{0} w_0\frac{\partial w_0}{\partial h}dz\\ &\quad -a(\nabla h)^2\left\{\int_{-h}^{0} w_0\frac{\partial^2 w_0}{\partial h^2}dz+w_0\frac{\partial w_0}{\partial h}\bigg|_{z=-h}\right\}\\ &\quad +\left\{\nabla^2\int_{-h}^{0} w_0\psi\,dz-\int_{-h}^{0} w_0\nabla^2\psi\,dz\right\}-\nabla\psi.\nabla h w_0|_{z=-h}. \quad \text{(A 1)}\end{aligned}$$

The integrals on the left-hand side of (A 1) have the values p and $-\omega^2 q/\lambda_0$ respectively. Hence the mild-slope equation (2) can be derived if we can justify the neglect of the right-hand-side 'forcing' terms in (A 1). The required estimates differ according to the local depth of the water. It has already been noted in § 2 that for shallow water the validity of the mild-slope equation should not depend upon the actual slope, so we shall concentrate here upon the case in which the water depth is comparable to the local wavelength. If the seabed slope is small, $O(\epsilon)$ say, then relative to the left-hand-side terms in (A 1) the 'forcing' terms are of orders ϵ^2 and $\epsilon|\psi|/|a|$. Thus it remains to estimate the magnitude of the non-propagating part ψ of the velocity potential. One means of doing this is to represent ψ by a superposition of eigenmodes:

$$\psi=\sum_{j=1}^{\infty} a_j(\mathbf{x})\,w_j(z;h(\mathbf{x})).$$

Taking the w_j component of the equations of motion we can derive an equation for a_j of the same form as (A 1) but with the important difference that the λ_j

are all positive. This difference means that for the non-propagating modes the size of a_j can be estimated from the local magnitude of the 'forcing' terms (the name non-propagating is due to this localness property). Thus we can estimate that $|\psi| = O(\epsilon|a|)$, and consequently the mild-slope equation (2) agrees with (A 1) to order ϵ^2.

For the propagating mode an equation error $O(\epsilon^2)$ does not imply that the solutions are accurate to $O(\epsilon^2)$. As the waves propagate over a linear distance $O(\epsilon^{-1})$, in which the depth varies, the cumulative effect of phase errors could lead to errors in the solution $O(\epsilon)$. Furthermore, in the main text the wave height ζ was assumed to satisfy the mild-slope equation, yet we have only derived the equation for the w_0 component of the velocity potential. For periodic waves the wave height is directly proportional to the value at the mean free surface of the total velocity potential. Thus the actual wave height can be expected to differ from the solution of the mild-slope equation by $O(\epsilon)$, the two sources of error being the presence of modes other than w_0 and the intrinsic error in the mild-slope equation as applied to the w_0 mode.

Appendix B. The solution near a beach

If we are to obtain a series solution of (6) about the regular singular point $r = a$, then our first task is to obtain a series solution in powers of $x = (r-a)/a$ for the coefficients p and q. After a straightforward calculation we find from (3) that

$$p\omega^2/g^2 = u - \tfrac{2}{3}u^2 + \tfrac{8}{45}u^3 - \tfrac{8}{945}u^4 + \dots,$$
$$q = 1 - \tfrac{1}{3}u + \tfrac{2}{45}u^2 + \tfrac{8}{945}u^3 + \dots,$$

where $u = \omega^2 h/g$ (cf. figure 1). For the special case of a conical island with beach slope α we have $u = ax(\omega^2\alpha/g)$, so the above expansions can be converted trivially into Taylor series with respect to x.

The required series solution is written as

$$\zeta = 1 + \gamma_1 x + \gamma_2 x^2 + \gamma_3 x^3 + \dots .$$

Substitution into the differential equation, equating coefficients and re-arranging yields

$$\gamma_1 = -(\omega^2 a/\alpha g),$$
$$\gamma_2 = \tfrac{1}{4}[m^2 - \gamma_1 + (1-\alpha^2)\gamma_1^2],$$
$$\gamma_3 = -\tfrac{1}{9}[\{10 - (5 - \tfrac{4}{3}\alpha^2)\gamma_1\}\gamma_2 + \gamma_1\{1 - (3 - 2\alpha^2)\gamma_1 + (1 - \tfrac{2}{3}\alpha^2 + \tfrac{4}{5}\alpha^4)\gamma_1^2\}].$$

This regular series suffices since the singular solution is inappropriate to the physical problem.

REFERENCES

COHEN, A. M. & JONES, D. E. 1974 A note on the numerical solution of some second order differential equations. *J. Inst. Math. Appl.* **13**, 379–384.

ECKART, C. 1950 The ray particle analogy. *J. Mar. Res.* **9**, 139–144.

ECKART, C. 1951 Surface waves on water of variable depth. *Scripps Inst. Oceanog., Lecture Notes, Fall Semester* 1950–51, *Ref.* 51–12, *Wave Rep.* no. 100.

GILL, S. 1951 A process for the step-by-step integration of differential equations in an automatic digital computing machine. *Proc. Camb. Phil. Soc.* **47**, 96–108.

HOMMA, S. 1950 On the behaviour of seismic sea waves around a circular island. *Geophys. Mag.* **21**, 199–208.

KELLER, J. B. 1958 Surface waves on water of non-uniform depth. *J. Fluid Mech.* **4**, 607–614.

KELLER, J. B. & RUBINOW, S. I. 1960 Asymptotic solution of eigenvalue problems. *Ann. Phys.* **9**, 24–75. (Errata, pp. 303–305.)

LAUTENBACHER, C. 1970 Gravity wave refraction by islands. *J. Fluid Mech.* **41**, 655–672.

LONGUET-HIGGINS, M. S. 1967 On the trapping of wave energy round islands. *J. Fluid Mech.* **29**, 781–821.

MORSE, P. M. & FESHBACH, H. 1953 *Methods of Theoretical Physics.* McGraw-Hill.

NEWMAN, J. N. 1965 Propagation of water waves over an infinite step. *J. Fluid Mech.* **23**, 399–415.

SHEN, M. C., MEYER, R. E. & KELLER, J. B. 1968 Spectra of water waves in channels and around islands. *Phys. Fluids*, **11**, 2289–2304.

SMITH, R. 1974 Edge waves on a beach of mild slope. *Quart. J. Mech. Appl. Math.* **27**, 102–110.

SUMMERFIELD, W. 1972 Circular islands as resonators of long-wave energy. *Phil. Trans.* A **272**, 361–402.

URSELL, F. 1952 Edge waves on a sloping beach. *Proc. Roy. Soc.* A **214**, 79–97.

VASTANO, A. C. & REID, R. O. 1967 Tsunami response for islands: verification of a numerical procedure. *J. Mar. Res.* **25**, 129–139.